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An Analytic Formula for Heating Due to Ozone Absorption

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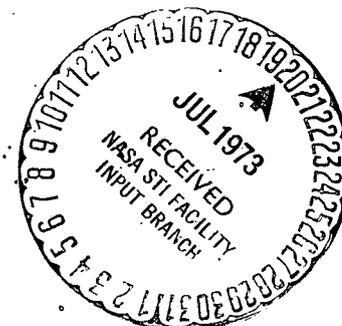
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From the tropopause to about 80 km, the major source of radiative heating in the atmosphere is the absorption of sunlight by ozone. This absorption occurs in the Hartley (2000 Å - 3000 Å), Huggins (above 3000 Å) and Chappuis (5000 Å - 7000 Å) bands (Craig, 1965, p. 168; also Craig, 1950) and is only slightly temperature and pressure dependent. Consequently, to a high degree of approximation one may write

$$I_{\nu} = I_{\infty} e^{-K_{\nu} u} \quad (1)$$

and

$$Q = \int_{\nu} K_{\nu} I_{\nu} n d\lambda \quad (2)$$

where

$$Q = \text{heating in ergs/sec cm}^3$$

$$u = \int_{\infty}^z n dz / \cos \theta \quad 4$$

where

u = optical depth in cm NTP

θ = zenith angle

n = ozone density in (cm NTP/cm)

K_{ν} = absorption coefficient (cm NTP)⁻¹

I_{∞} = insolation (incident solar intensity) (ergs/cm² sec Å)

I_{ν} = intensity (ergs/cm² sec Å)

λ = wavelength in Å

z = height (cm)

⁴ This expression is appropriate only for a flat atmosphere and must be modified to take sphericity into account.

Distributions of I_{∞} and κ_{ν} are shown in Figs. 1-3. The quantity $Q/n \equiv \eta$ is solely a function of u , and for the choice of spectral data shown in Figs. 1-3 leads to the distribution of η with u shown in Fig. 4. As noted in Lindzen and Goody (1965) there are two regions of the atmosphere where η may be approximated by a constant:

i) above about 4-5 km where $u < 2 \times 10^{-3}$ cm NTP we are in a region which is almost transparent for all ozone bands.

Hence, ozone band radiation is not significantly attenuated and the heating is largely due to ozone's strongest band, the Hartley band

ii) below about 30 km where 0.3 cm NTP $> u > 0.15$ cm NTP, most radiation in the Hartley and Huggins bands has been absorbed but the atmosphere is still almost transparent for the Chappuis which dominates the heating.⁵

The question now arises as to whether a simple expression exists which describes both regions of "constant" η as well as the transition between them. We, moreover, wish a convenient way of relating parameters in the expression to spectral data. Now, in the regions of constant η , η depends only on average values of spectral intensities and absorption coefficients for the Hartley and Chappuis bands.

Thus a simple solution might be to use average values for I_{ν} and κ_{ν} ,

⁵N. B. Figure 4 is misleading on this matter since it includes values of u which greatly exceed the atmosphere's total ozone content; such values could be important, however, in time dependent calculations where Θ may approach $\pi/2$.

appropriate to the Hartley and Chappuis bands, in (1) and (2) modelling both bands as simple Chapman layers (Craig, 1965, pp. 147-150). One then obtains for η the following expression

$$\eta = I_H \kappa_H \Delta\lambda_H e^{-\kappa_H u} + I_C \kappa_C \Delta\lambda_C e^{-\kappa_C u} \quad (3)$$

where I_H, κ_H and I_C, κ_C are average I, κ for Hartley (_H) and Chappuis (_C) bands and $\Delta\lambda_H, \Delta\lambda_C$ are the bandwidths taken for these bands. For the range of u 's relevant to the earth's atmosphere, κ_H, I_H, κ_C and I_C can be chosen to accurately model the regions of constant η ; (3) will then also describe the transition between the two regions. Unfortunately, the accuracy of such an approximation proved poor in the transition zone because of the importance of the Huggins bands in that zone. Modelling the Huggins bands proved reasonably simple. From Fig. 2 we see that κ_v is almost a linear function on a semi-log plot. Thus we may write

$$\kappa_v = \kappa_{Hu} e^{-M\lambda} \quad (4)$$

κ_{Hu}, M are constants of the straight line on the semi-log plot. Taking for I_v , an average over the Huggins bands, I_{Hu} , we can easily integrate (2) obtaining

$$\eta_{Hu} = \frac{I_{Hu}}{M \cdot u} \left[e^{-u\kappa_{Hu}} e^{-m\lambda_{LONG}} - e^{-u\kappa_{Hu}} e^{-m\lambda_{SHORT}} \right] \quad (5)$$

where λ long and λ short are the wavelength limits used for the Huggins bands.

Thus an improved approximation to η consists in the sum of (3) and (5), namely

$$\eta = I_H \kappa_H \Delta\lambda_H e^{-\kappa_H u} + I_c \kappa_c \Delta\lambda_c e^{-\kappa_c u} + \frac{I_{Hu}}{M \cdot u} \left[e^{-u\kappa_{Hu}} e^{-m\lambda_{LONG}} - e^{-u\kappa_{Hu}} e^{-m\lambda_{SHORT}} \right] \quad (6)$$

Expression (6) has, in principle, a substantial number of adjustable parameters: the widths of the Hartley, Huggins and Chappuis bands, I_H , I_c , I_{Hu} , and κ_H and κ_c . If one merely estimates these parameters on the basis of Figs. 1-3, we obtain a uniform 20 percent accuracy in approximating Fig. 4. However, with a little experimentation we were able to choose these parameters so as to obtain the approximation shown in Fig. 4 which, in general, has about a 5 percent accuracy. The parameters chosen are shown in Figs. 1-3. The following formulae should prove useful when dealing with more recent or better data than that shown in Figs. 1-3.

Hartley bands:

$$\begin{aligned}
 I_H &= 9 \text{ ergs/cm}^2 \text{ sec } \text{\AA} \\
 \kappa_H &= 260 \text{ cm NTP}^{-1} = 80\% (\kappa_v \text{ maximum for Hartley}) \\
 \Delta\lambda_H &= 375 \text{ \AA}
 \end{aligned}
 \tag{7}$$

Chappuis bands:

$$\begin{aligned}
 I_C &= 180 \text{ ergs/cm}^2 \text{ sec } \text{\AA} \\
 \kappa_C &= 0.118 \text{ cm NTP}^{-1} = 80\% (\kappa_v \text{ maximum for Chappuis}) \\
 \Delta\lambda_C &= 1650 \text{ \AA}
 \end{aligned}
 \tag{8}$$

Huggins bands:

$$\begin{aligned}
 \kappa_{Hu} &= 1.99 \times 10^{17} \\
 M &= 0.0126 \\
 I_{Hu} &= 53 \text{ ergs/cm}^2 \text{ sec } \text{\AA} \\
 \lambda_{\text{SHORT}} &= 2750 \text{ \AA} \\
 \lambda_{\text{LONG}} &= 3400 \text{ \AA}
 \end{aligned}
 \tag{9}$$

Although our formulae for the Huggins bands are not explicitly in a form which would permit adaptation to other data, the method of choosing the parameters, discussed earlier in this note, is obviously adaptable. We chose to model Craig's (1951) somewhat outdated calculation simply because complete spectral information was conveniently available for his calculation. This case is sufficient to demonstrate that ozone heating can be accurately modelled by two Chapman layers plus a modified Chapman layer for the Huggins bands, which is our main result.

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FIGURE LEGENDS

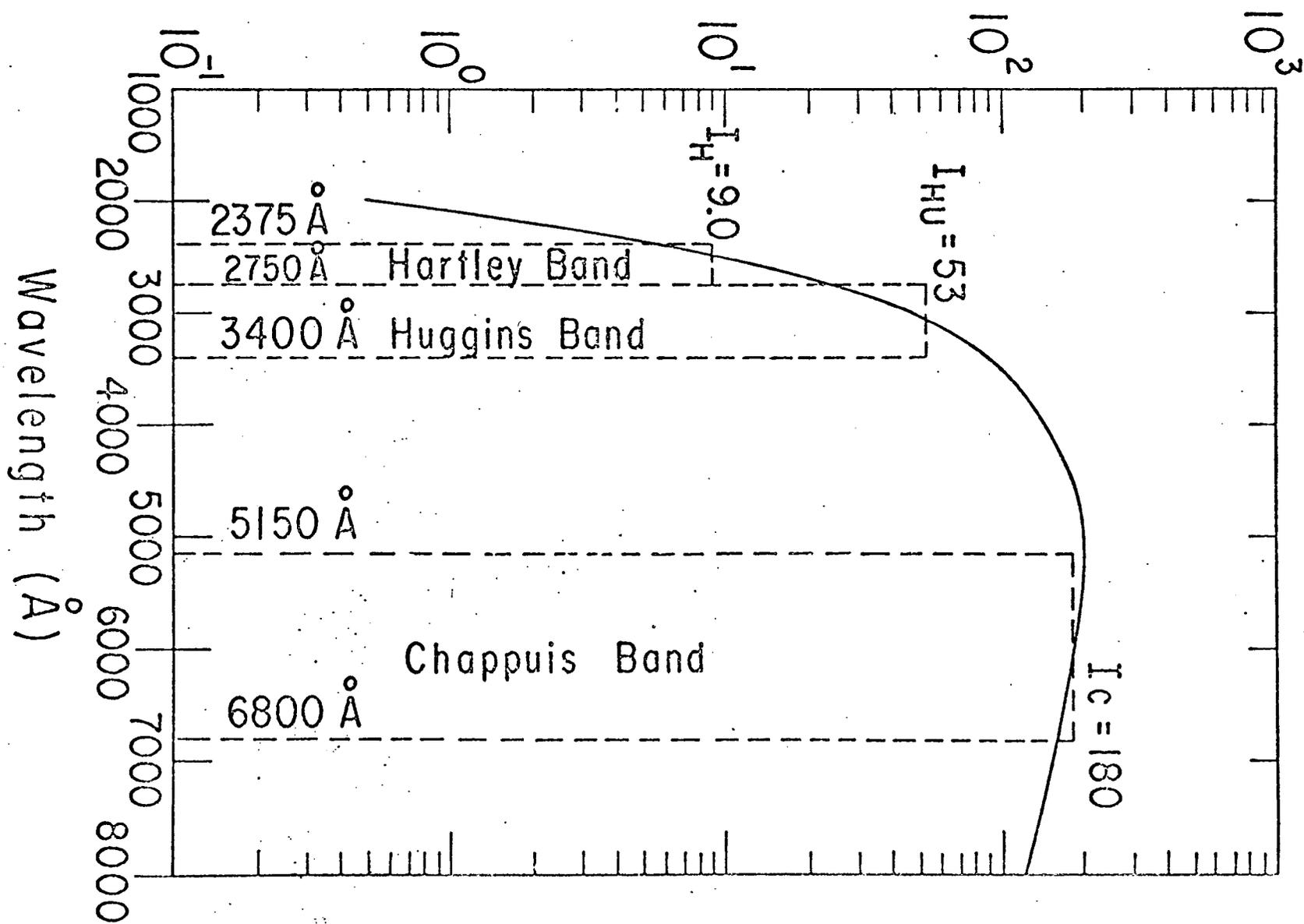
Fig. 1. The solid curve, taken from Craig (1950), shows the spectral distribution of incoming solar radiation. Also shown are the average values of intensity and band limits used in our modelling procedure.

Fig. 2. The solid curve shows absorption by ozone as a function of wavelength for the Hartley and Huggins bands (as taken from Craig, 1950). Also shown (dashed curves) are the approximations to the absorption used in our modelling procedure.

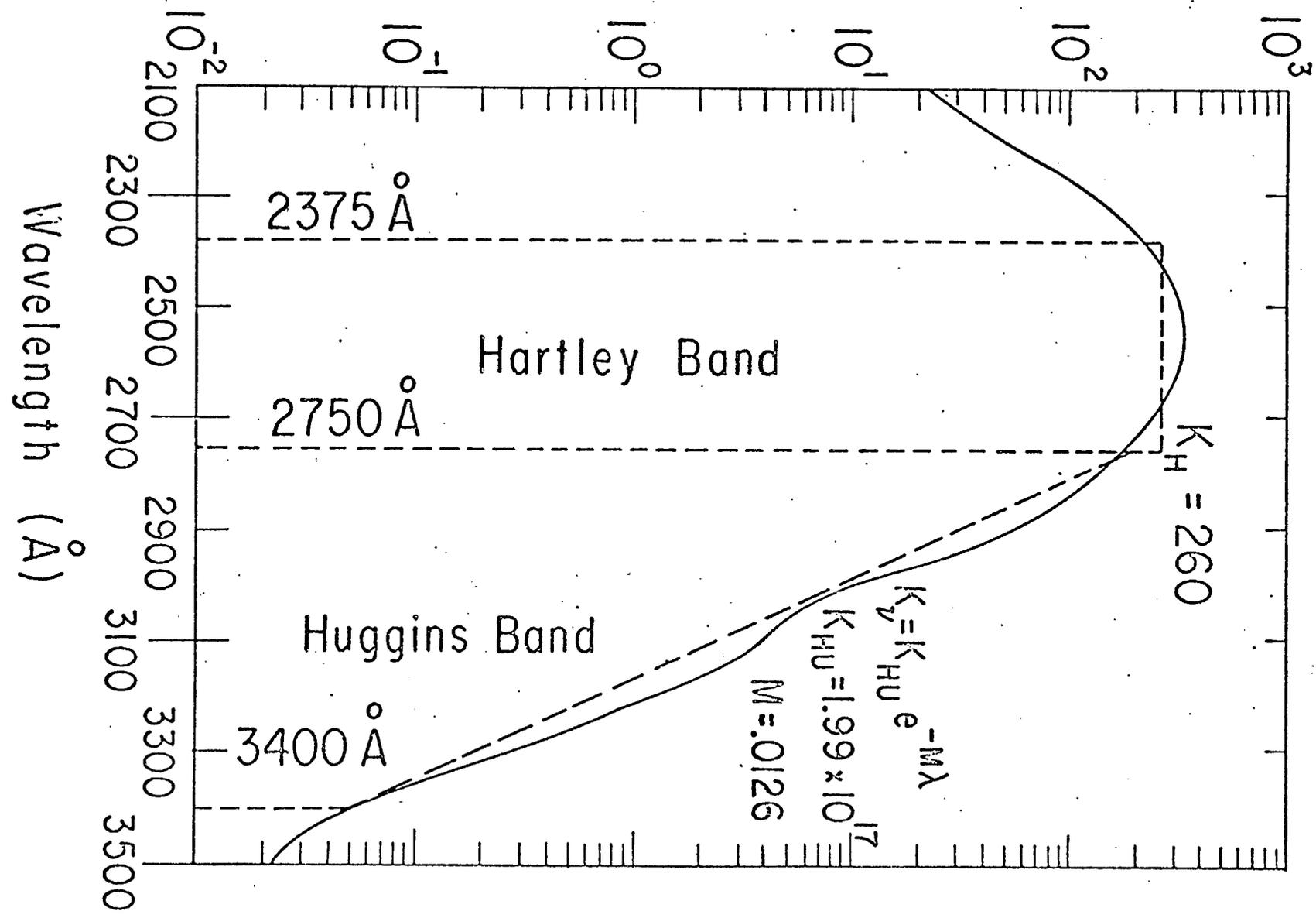
Fig. 3. Same as Fig. 2 but for Chappuis band.

Fig. 4. Specific heating, η , due to ozone as a function of optical depth, u . Solid curve represents Craig's (1951) calculation; dashed curve represents our analytic formula.

Incoming Solar Radiation ($\text{ergs cm}^{-2} \text{sec}^{-1} \text{\AA}^{-1}$)



Absorption Coefficient (cm NTP⁻¹)



10

Absorption Coefficient (cm NTP⁻¹)

